

Review for Exam 1

1) Find the infinite sum

(a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots +$

(b) $\left(\frac{4}{5}\right)^3 + \left(\frac{4}{5}\right)^4 + \left(\frac{4}{5}\right)^5 + \left(\frac{4}{5}\right)^6 + \dots +$

(c) $1 + \left(\frac{2}{7}\right)^1 + \left(\frac{2}{7}\right)^2 + \left(\frac{2}{7}\right)^3 + \left(\frac{2}{7}\right)^4 + \dots +$

(d) $1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^8 + \dots +$

(e) $\left(\frac{1}{5}\right)^1 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)^5 + \left(\frac{1}{5}\right)^7 + \left(\frac{1}{5}\right)^9 + \dots +$

(f) If n is a non negative integer and x a real number, what should be the sum

$$x^n + x^{n+1} + x^{n+2} + x^{n+3} + x^{n+4} + x^{n+5} + \dots + ?$$

Does the formula you've obtained ever seem to lead to unintuitive results?

Explain.

(g) $2^2 + 2^3 + 2^4 + 2^5 + 2^6 + \dots +$

(h) $1 + 1^2 + 1^3 + 1^4 + 1^5 + 1^6 + \dots +$

(i) $(-1) + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6 + \dots +$

2) Express the decimal as a fraction

(a) 0.2

(b) $0.22222 \dots = 0.\overline{2}$

(c) $0.838383 \dots = 0.\overline{83}$

(d) $0.213213213 \dots = 0.\overline{213}$

(e) $0.233333 \dots = 0.2\overline{3}$

3) A rubber ball has the property that, when it is dropped to the ground from height h , it bounces back to height rh , where r is a fraction. How many times will the ball bounce off the ground? Find the total distance the ball will travel if dropped from height h . Use the result obtained to calculate:

(a) when $h = 10$ and $r = 1/2$

(b) when $h = 15$ and $r = 2/5$

4) Two runners start off on opposite sides of the track at a distance of 15 miles. Runner A is running toward Runner B at the rate of 3 mi/hr relative to the track. Runner B is moving toward A at the rate of 2 mi/hr (relative to the track). If a fly is flying back and forth from the nose of one runner to another at 12 mi/hr until the runners meet, how much distance is covered by the pesky creature? How many back-and-forth trips does the fly make?

(a) Draw a picture and label the variables of the problem.

(b) Can you solve this problem for any d = length of the track, a = speed of Runner A, b = speed of Runner B, f = speed of the fly, and D = total distance traveled by the fly?

5) Explain why the area of a rectangle of length a and height b is ab .

6) Explain why the area of a right triangle of length a and height b is $\frac{ab}{2}$. Can you see why the formula still holds true if the triangle is no longer assumed to be right?

7) Prove (as we have done in class) that in a right triangle with sides a , b , and hypotenuse c , these quantities are related by $a^2 + b^2 = c^2$.

8) Let m be an integer. True or False? Justify your answer! Be careful!

(a) $5|m^2 \rightarrow 5|m$

(b) $5|m^5 \rightarrow 5|m^3$

(c) $4|m^5 \rightarrow 4|m$

(d) $4|m^5 \rightarrow 4|m^3$

(e) $6|m^2 \rightarrow 6|m$

(f) $14|m^5 \rightarrow 14|m^3$

(g) $14|m^5 \rightarrow 8|m^3$

9) Show that the following numbers are irrational (i.e. cannot be expressed as a ratio of two integers):

(a) $\sqrt{2}$

(b) $\sqrt{5}$

(c) $\sqrt{7}$

(d) $\sqrt{12}$

10) Recall that we defined \mathbb{Q} as the set of all fractions (equivalently, as the set of all finite sums of integer rations) and \mathbb{R} , the real numbers, as the set of all numbers expressible as an infinite sum of nonzero fractions. Using this definition, decide which of the following are rational numbers and which are real:

(a) 0.5

(b) $0.\bar{9}$

(c) 2

(d) 0

(e) -1

(f) $1 + \left(-\frac{2}{7}\right)^1 + \left(-\frac{2}{7}\right)^2 + \left(-\frac{2}{7}\right)^3 + \left(-\frac{2}{7}\right)^4 + \dots +$

(g) $\frac{1}{3}$

(h) $\frac{m}{n}$, where m and n are integers; $n \neq 0$

11) Let $A = \{1, 2, 3\}$, $B = \{1, 2, 4\}$, $C = \{2, 3, \{2, 4\}, \emptyset, \{1, 2, 3\}\}$

(a) Compute $A \cup B$, $A \cup C$, $A \cup B \cup C$.

(b) Compute $A \cap B$, $A \cap C$, $A \cap B \cap C$.

(c) Compute $\mathcal{P}(A)$ and $|\mathcal{P}(C)|$.

(d) True or False? $\emptyset \subset A$

(e) True or False? $\emptyset \subset C$

(f) True or False? $\emptyset \in A$

(g) True or False? $\emptyset \in C$

(h) True or False? $A \subset C$

(i) True or False? $A \in C$

(k) True or False? $B \cap C = \{2, 4\}$

(j) Compute $B \cap \{\{1, 2, 4\}\}$